Probabilistic Quantum Clone via GHZ States

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We propose a probabilistic quantum clone scheme using GHZ states, Bell basis measurement, single-qubit unitary operations and generalized measurement, all of which are within the reach of current technology. Compared to another possible scheme via Tele-CNOT gate (Gottesman and Chuang, *Nature* **402,** 390 (1999)), the present scheme may be used in experiment to *clone the states of one particle to those of two different particles* with higher probability and less GHZ resources.

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I. INTRODUCTION

Quantum computers can solve problems that classical computers can never solve $[1-2]$. However, the practical implementation of such device need careful consideration of the minimum resource requirement and feasibility of quantum operation. The basic operation in quantum computer is unitary evolution, which can be performed using some single-qubit operations and Controlled-NOT (CNOT) gates [3]. While single-qubit operation can be executed easily [4], the realization of CNOT operations between two particles (for example two photons) encounters great difficulty in experiment [5]. With linear optical devices (such as beam splitters, phase shifters, etc.), the CNOT operations between the several quantum qubits (such as locations and polarization) of a single photon is within the reach of current quantum optics technology [6], but nonlinear interactions are required for the construction of practical CNOT gate of two particles [5]. Those nonlinear interactions are normally very weak, which forecloses the physical realization of quantum logic gate.

To solve this problem, Gottesman and Chuang [7] suggested that a generalization of quantum teleportation [8] −using single-qubit operations [4], Bell-basis measurements $[9-12]$ and certain entangled quantum states such as Greenberger-Horne-Zeilinger (GHZ) states [13] −is sufficient to construct a universal quantum computer. They presented systematic constructions for an infinite class of reliable quantum gate (including Tele-CNOT gate). Experimentally, quantum teleportation has been realized $[14-16]$ and three-photon GHZ entanglement was observed [17]. Thus their construction of quantum gates offers possibilities for relaxing experimental constraints on realizing quantum computers.

Unfortunately, up to now there has been no way to experimentally distinguish all four of the Bell states, although some schemes do work for two of the four required cases − yielding at most a 50% absolute efficiency $[9-12]$. In Gottesman and Chuang's scheme, two GHZ states and three Bell-basis measurements are needed to perform a CNOT operation, which yields 1/8 probability of success in experiment. To complete a unitary operator, many CNOT gates may be needed, which make the probability of success close to zero. Moreover, the creation efficiency of GHZ states is still not high in experiment now [17]. Therefore a practical experiment protocol requires careful consideration of the minimum resource and the maximum probability of success.

In this paper we will investigate the problem of probabilistic quantum clone with GHZ states, Bell basis measurement, single-qubit unitary operation and generalized measurement (it can be performed with location qubit of the photon as the probe [6]). Consider a sender Alice holds an one-qubit quantum state $|\phi\rangle$ and wishes to transmit identical copies to N associates (Bob, Claire,

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etc.). Quantum no-cloning theorem [18] implies that the copies cannot be perfect; but this result does not prohibit cloning strategies with a limited degree of success. Two most important cloning machines−universal and state-dependent (include deterministic, probabilistic and hybrid)−have been proposed by some authors [19 − 27]. It is not available [28] for Alice to generate the copies locally using an appropriate quantum network [20, 22, 26, 27] and then teleport each one to its recipient by means of teleportation due to the difficulty of executing CNOT operation [5]. Recently Murao *et al.* [29, 30] presented an optimal 1 to N universal quantum teleclone strategy via a $(2N)$-particle entangled state. Such entanglement is difficult to prepare in experiment when N is large. Quantum probabilistic cloning machine is designed to reproduce perfectly linear independent states secretly chosen from a finite set with no-zero probability [25 − 27]. Of course the corresponding teleprocess can be performed via the Tele-CNOT gates [7] according to the cloning strategies provided in [26, 27]; but such procedure needs too many GHZ states and Bell-basis measurements and can succeed with probability close to zero. The scheme we propose in this paper needs only $(N-1)$ GHZ states and $(N-1)$ Bell-basis measurements to implement $M \to N$ clone. Although such process can not reach the optimal probability as that in local situation, it may be used in current experiment to *clone the states of one particle to those of two different particles* with higher probability.

The rest of the paper is organized as follows. In section II we discuss some strategies of probabilistic clone and present the concept of *probability spectrum* to describe different strategies. Comparing two most important ones, we show that M entries $1 \to N$ clone give more copies at the price of higher probability of failure than one $M \to N$ clone. In Section III, we present the probabilistic telecloning process using three-particle entangled state and also show how to construct the entangled state from GHZ state via local operations and classical communication. A summary is given in Section IV.

II. STRATEGIES OF PROBABILISTIC CLONE

Generally, the most useful states are $|\phi_\pm(\theta)\rangle = \cos\theta\,|1\rangle \pm \sin\theta\,|0\rangle$ in quantum information theory. Given M initial copies, Alice need not to always execute the cloning operation by taking these copies as a whole. Suppose Alice divides the M copies into m different kinds of shares, each of which includes ϑ_i entries $k_i \to N_i$ cloning processes. These parameters should satisfy

$$\sum_{i=1}^{m} k_i\vartheta_i = M. \tag{2.1}$$

The probability of obtaining x copies for Alice can be represented as

$$P(x) = \sum_{\sum_{i=1}^{m} g_i N_i = x} \prod_{i=1}^{m} C_{\vartheta_i}^{g_i} \gamma_{k_i N_i}^{g_i} \left(1-\gamma_{k_i N_i}\right)^{\vartheta_i - g_i} \tag{2.2}$$

where $C_{\vartheta_i}^{g_i} = \vartheta_i!/g_i!(\vartheta_i - g_i)!$, g_i denotes successful cloning attempts in ϑ_i same items, $\gamma_{k_i N_i}$ is the success probability of $k_i \to N_i$ clone, which is

$$\gamma_{k_i N_i} = \frac{1-\cos^{k_i} 2\theta}{1-\cos^{N_i} 2\theta}. \tag{2.3}$$

$P(x)$ is the discrete function of x and can be represented as a series of discrete lines in the $P(x) - x$ plane, which we called *Probability Spectrum*. Different probabilistic cloning strategies are corresponding to different *Probability Spectrums*.

Two important parameters can be obtained from *Probability Spectrum*, that is, the expected value of the output copies E and the probability of failure F, which are defined as

$$E\{k_i, N_i, \vartheta_i\} = \sum_{x=0}^{\sum_{i=1}^{m}\vartheta_i N_i} xP(x), \tag{2.4}$$

$$F\{k_i, N_i, \vartheta_i\} = \sum_{x=0}^{N-1} P(x). \tag{2.5}$$

It is regarded as failure if the copies number Alice attains less than N. When M is large, above two parameters can well describe different cloning strategies. In the following, we discuss two most important cloning strategies:

(1) cloning the M copies as a whole $(M \to N)$.

(2) cloning each copy respectively ($M\times(1\to N)$).

The second is included for it is the strategy we choose in probabilistic telecloning process. Compare above two strategies with the two parameters E and F, we find the second give more copies at the price of higher probability of failure. In fact, if Alice choose the second strategy, the clone attempt may succeed for two or more initial copies and thus Alice may have chance to get more than N copies. The expected value of the two different strategies can be represented as

$$E_1 = N\gamma_{MN}, \tag{2.6}$$

$$\begin{aligned} E_2 &= \sum_{k=0}^{M} kNC_M^k\gamma_{1N}^k\left(1-\gamma_{1N}\right)^{M-k} \\ &= MN\gamma_{1N}\sum_{k=1}^{M} C_{M-1}^{k-1}\gamma_{1N}^{k-1}\left(1-\gamma_{1N}\right)^{(M-1)-(k-1)} \\ &= MN\gamma_{1N}, \end{aligned} \tag{2.7}$$

where $2\le M<N$. Denote $t=\cos 2\theta$, we get $\Delta E = E_2 - E_1 = N\widetilde{\Delta E}/\left(1-t^N\right)$, where $\widetilde{\Delta E} = M - Mt - 1 + t^M$. Obviously $0\le t\le 1$, if $t=0$, $\widetilde{\Delta E} = M-1>0$; if $t=1$, $\gamma_{MN}=\frac{M}{N}$, $\Delta E = \widetilde{\Delta E} = 0$. When $t\ne 1$, $\frac{d\widetilde{\Delta E}}{dt} = -M + Mt^{M-1} < 0$, thus $\widetilde{\Delta E}$ is monotonously decreasing and always greater than zero, which means

$$E_1 \le E_2. \tag{2.8}$$

The equality is satisfied only when $|\varphi_+(\theta)\rangle = |\varphi_-(\theta)\rangle$ $(t\ne 1)$. ΔE is very large when M is large. The expected value for different M, N are plotted in Figure 1.

Figure 1

The failure probabilities of above two strategies are

$$F_1 = 1-\gamma_{MN} = t^M\left(1-t^{N-M}\right)/\left(1-t^N\right), \tag{2.9}$$

$$F_2 = \left(1-\gamma_{1N}\right)^M = \left(\left(t-t^N\right)/\left(1-t^N\right)\right)^M, \tag{2.10}$$

respectively. Note the fact that $\left(\prod_{i=1}^n a_i\right)^{\frac{1}{M}} \le \frac{1}{M}\left(\sum_{i=1}^n a_i\right)$, for any $a_i\ge 0$ (equality can be satisfied if and only if $a_1=a_2=...=a_n$), we can write

$$\begin{aligned} F_1 &= \frac{t^M}{\left(1-t^N\right)^M}\left(1-t^N\right)^{M-1}\left(1-t^{N-M}\right) \\ &\le \frac{t^M}{\left(1-t^N\right)^M}\left(1-\frac{t^{N-M}+(M-1)t^N}{M}\right)^M \\ &\le \frac{t^M}{\left(1-t^N\right)^M}\left(1-t^{N-1}\right)^M = F_2. \end{aligned} \tag{2.11}$$

where $F_1=F_2$ is satified if and only if $t=0$ or 1, which means $\theta=\pi/4$ or 0. The failure probabilities for different M, N are illustrated in Figure 2.

Figure 2

Now that the two different strategies have both advantage and shortage, Alice should choose one according to her need. If she need more copies, she can adopt $1\to N$ strategy. If she wishes to obtain the copies with greater success probability, she should choose $M\to N$ cloning process.

III. PROBABILISTIC TELECLONING PROCESS

Suppose Alice holds M copies of one-qubit quantum state $|\phi\rangle_X$ that is secretly chosen from a set $\{|\phi_\pm(\theta)\rangle = \cos\theta\,|1\rangle \pm \sin\theta\,|0\rangle\}$ and wishes to clone them to N associates (Bob, Claire, etc.). In local situation, she can do so using a unitary-reduction operation on the $N+1$ qubit (N qubit of cloning system and a probe to determine whether the clone is successful) with maximum success probability [26] $\gamma_{MN} = \left(1-\cos^M 2\theta\right)/\left(1-\cos^N 2\theta\right)$. This unitary-reduction operator can be decomposed into the interaction between two particles using a special unitary gate [26]:

$$D\left(\theta_1,\theta_2\right)|\phi_\pm(\theta_3)\rangle\,|1\rangle = |\phi_\pm(\theta_1)\rangle\,|\phi_\pm(\theta_2)\rangle\,. \tag{3.1}$$

with $\cos 2\theta_3 = \cos 2\theta_1\cos 2\theta_2$ and $0\le\theta_j\le\pi/4$, which suffices to determine θ_3 uniquely. This operation transfers the information describing the initial states $|\phi_\pm(\theta_1)\rangle\,|\phi_\pm(\theta_2)\rangle$ into one qubit $|\phi_\pm(\theta_3)\rangle$. With such pairwize interaction, the information of the initial states $|\phi_\pm(\theta)\rangle^{\otimes M}$ may be transferred into one qubit $|\phi_\pm(\theta_M)\rangle\,|0\rangle^{\otimes(M-1)}$ using corresponding operator $D_M = D_1\left(\theta_{M-1},\theta_1\right)D_2\left(\theta_{M-2},\theta_1\right),...D_{M-1}(\theta_1,\theta_1)$, where $D_j\left(\theta_{M-j},\theta_1\right)$ is denoted as the operator $D\left(\theta_{M-j},\theta_1\right)$ acts on particles $(1,j+1)$ and θ_j is determined by $\cos 2\theta_M = \cos^j 2\theta$. This operator is unitary and D_M^+ can implement the reversible transformation. Thus we only need to transfer the state $|\phi_\pm(\theta_M)\rangle$ to the appropriate form $|\phi_\pm(\theta_N)\rangle$ to obtain $|\phi_\pm(\theta)\rangle^{\otimes N}$ using operation D_N^+ (with similar definition as D_M^+). This process can be accomplished by a unitary-reduction operation

$$U\left|\phi_{\pm}\left(\theta_{M}\right)\right\rangle\left|P_{0}\right\rangle=\sqrt{\gamma}\left|\phi_{\pm}\left(\theta_{N}\right)\right\rangle\left|P_{0}\right\rangle+\sqrt{1-\gamma}\,|1\rangle\left|P_{1}\right\rangle. \quad (3.2)$$

U is unitary and the transformation probability $\gamma=\gamma_{MN}$. U is a qubit 1 controlling probe P rotation $R_y(2\omega)=\begin{pmatrix}\cos\omega & \sin\omega\\ -\sin\omega & \cos\omega\end{pmatrix}$ with $\omega=\arccos\sqrt{\frac{(1-\cos^M 2\theta)(1+\cos^N 2\theta)}{(1+\cos^M 2\theta)(1-\cos^N 2\theta)}}$ and can be accomplished in experiment by choosing location qubit of the photon as the probe qubit [6].

Operations D_M and D_N^+ involve the interactions of two photons which is difficult to implement in current experiment. In this paper, we adopt $M\times(1\to N)$ strategy to substitute D_M and transfer M copies of the states $|\phi_\pm(\theta)\rangle$ to $|\phi_\pm(\theta_N)\rangle$ respectively using similar unitary-reduction operations as that in Eq. (3.2). To substitute the operation D_N^+, we may use three-particle entanglement to implement the operator $D_j(\theta_{N-j},\theta_1)$, which acts as

$$D_j\left(\theta_{N-j},\theta_1\right)\left|\phi_\pm\left(\theta_{N-j+1}\right)\right\rangle|0\rangle=\left|\phi_\pm\left(\theta_{N-j}\right)\right\rangle\left|\phi_\pm\left(\theta_1\right)\right\rangle. \quad (3.4)$$

Assume Alice and the j-th associate C_j share a three-particle entangled state $\left|\psi^j\right\rangle_{SAC_j}$ as a starting resource. This state must be chosen so that, after Alice performs local Bell measurements and informs C_j of the results, she and C_j can obtain the state $\left|\phi_\pm\left(\theta_{N-j}\right)\right\rangle_A\left|\phi_\pm\left(\theta_1\right)\right\rangle_{C_j}$ by using only local operation. Denote $\left|\varphi_i^j\right\rangle=D_j\left(\theta_{N-j},\theta_1\right)|i\rangle|0\rangle$, $i\in\{0,1\}$, a choice of $\left|\psi^j\right\rangle_{SAC_j}$ with these properties may be the three-particle state

$$\left|\psi^j\right\rangle_{SAC_j}=\frac{1}{\sqrt{2}}\left(|0\rangle_S\left|\varphi_1^j\right\rangle_{AC_j}-|1\rangle_S\left|\varphi_0^j\right\rangle_{AC_j}\right), \quad (3.5)$$

where S represents a single qubit held by Alice, which we should refer to as the "port" qubit. The tensor product of $\left|\psi^j\right\rangle_{SAC_j}$ with the state $\left|\phi_\pm\left(\theta_{N-j+1}\right)\right\rangle_X=h_j|1\rangle\pm t_j|0\rangle$ ($h_j=\cos\theta_{N-j+1}$, $t_j=\sin\theta_{N-j+1}$) held by Alice is four-qubit state. Rewriting it in a form that singles out the Bell basis of qubit X and S, we get

$$\begin{aligned}&\left|\Omega^{\pm j}\right\rangle_{XSAC_j} \quad (3.6)\\ &=-\frac{1}{2}\left|\Psi^-\right\rangle_{XS}\left(h_j\left|\varphi_1^j\right\rangle_{AC_j}\pm t_j\left|\varphi_0^j\right\rangle_{AC_j}\right)\\ &+\frac{1}{2}\left|\Psi^+\right\rangle_{XS}\left(h_j\left|\varphi_1^j\right\rangle_{AC_j}\mp t_j\left|\varphi_0^j\right\rangle_{AC_j}\right)\\ &\pm\frac{1}{2}\left|\Phi^-\right\rangle_{XS}\left(t_j\left|\varphi_1^j\right\rangle_{AC_j}\pm h_j\left|\varphi_0^j\right\rangle_{AC_j}\right)\\ &\pm\frac{1}{2}\left|\Phi^+\right\rangle_{XS}\left(t_j\left|\varphi_1^j\right\rangle_{AC_j}\mp h_j\left|\varphi_0^j\right\rangle_{AC_j}\right)\end{aligned}$$

Where $\left|\Psi^\pm\right\rangle_{XS}=\frac{1}{\sqrt{2}}\left(|01\rangle_{XS}\pm|10\rangle_{XS}\right)$, $\left|\Phi^\pm\right\rangle_{XS}=\frac{1}{\sqrt{2}}\left(|00\rangle_{XS}\pm|11\rangle_{XS}\right)$ are the Bell basis of the two-qubit system $X\otimes S$. The telecloning process can now be accomplished by the following procedure.

(i) Alice performs a Bell-basis measurement of qubits X and S, obtaining one of the four results $\left|\Psi^\pm\right\rangle_{XS}$, $\left|\Phi^\pm\right\rangle_{XS}$.

(ii) Alice use different strategies according to different measurement results. If the result is $\left|\Psi^-\right\rangle_{XS}$, the subsystem AC_j is project precisely into the state $h_j\left|\varphi_1^j\right\rangle_{AC_j}\pm t_j\left|\varphi_0^j\right\rangle_{AC_j}=\left|\phi_\pm\left(\theta_{N-j}\right)\right\rangle_A\left|\phi_\pm\left(\theta_1\right)\right\rangle_{C_j}$. If $\left|\Psi^+\right\rangle_{XS}$ is obtained, we must perform $\sigma_z\otimes\sigma_z$ on system AC_j since $\left|\varphi_0^j\right\rangle_{AC_j}$ and $\left|\varphi_1^j\right\rangle_{AC_j}$ obey the following simple symmetry:

$$\sigma_z\otimes\sigma_z\left|\varphi_i^j\right\rangle_{AC_j}=(-1)^{i+1}\left|\varphi_i^j\right\rangle_{AC_j}. \quad (3.7)$$

With above operations, the states of system AC_j are transferred to $\left|\phi_\pm\left(\theta_{N-j}\right)\right\rangle_A\left|\phi_\pm\left(\theta_1\right)\right\rangle_{C_j}$, just as operation $D_j\left(\theta_{N-j},\theta_1\right)$ functions.

(iii) In the case one of the other two Bell states $\left|\Phi^\pm\right\rangle_{XP}$ is obtained, the corresponding states are entangled states. For example, if measurement result is $\left|\Phi^-\right\rangle_{XP}$, the remained states can be written as $\left|\alpha_\pm\right\rangle=\frac{\pm1}{\sin 2\theta_{N-j+1}}\left(\left|\phi_\pm\left(\theta_{N-j}\right)\right\rangle\left|\phi_\pm\left(\theta_1\right)\right\rangle-\cos 2\theta_{N-j+1}\left|\phi_\mp\left(\theta_{N-j}\right)\right\rangle\left|\phi_\mp\left(\theta_1\right)\right\rangle\right)$. Both of the two states lie in the subspace spanned by states $\left|\phi_+\left(\theta_{N-j}\right)\right\rangle\left|\phi_+\left(\theta_1\right)\right\rangle$, $\left|\phi_-\left(\theta_{N-j}\right)\right\rangle\left|\phi_-\left(\theta_1\right)\right\rangle$. The inner-products show that $\left|\alpha_\pm\right\rangle$ are orthogonal to $\left|\phi_\mp\left(\theta_{N-j}\right)\right\rangle\left|\phi_\mp\left(\theta_1\right)\right\rangle$. So they are entangled states unless $\left|\phi_+\left(\theta_{N-j}\right)\right\rangle\left|\phi_+\left(\theta_1\right)\right\rangle$ and $\left|\phi_-\left(\theta_{N-j}\right)\right\rangle\left|\phi_-\left(\theta_1\right)\right\rangle$ are orthogonal, which means $\left|\phi_\pm(\theta)\right\rangle$ are orthogonal. When $\left|\phi_\pm\left(\theta_1\right)\right\rangle$ are not orthogonal, Alice and Clair must disentangle the states to the needed states $\left|\phi_\pm\left(\theta_{N-j}\right)\right\rangle\left|\phi_\pm\left(\theta_1\right)\right\rangle$ simultaneously using only local operations and classical communication. Unfortunately, this process can not be deterministic although both $\left|\alpha_+\right\rangle\to\left|\phi_+\left(\theta_{N-j}\right)\right\rangle\left|\phi_+\left(\theta_1\right)\right\rangle$ and $\left|\alpha_-\right\rangle\to\left|\phi_-\left(\theta_{N-j}\right)\right\rangle\left|\phi_-\left(\theta_1\right)\right\rangle$ can be deterministically executed according to Nielsen theorem [31]. In fact, suppose there exists a process H to accomplish this using only local operations and classical communication, the evolution equation of the composite system of particles A, C and the local auxiliary particle G^A, G^C can be expressed as

$$H\left|\alpha_{\pm}\right\rangle\left|G_0^A\right\rangle\left|G_0^C\right\rangle = \sum_{i=1}^{h}\sum_{k=1}^{l}\sqrt{\eta_{ik}}\left|\phi_{\pm}\left(\theta_{N-j}\right)\right\rangle\left|\phi_{\pm}\left(\theta_1\right)\right\rangle\left|G_i^A\right\rangle\left|G_k^C\right\rangle. \tag{3.8}$$

H is a linear operation, thus we get

$$H\left|\phi_{\pm}\left(\theta_{N-j}\right)\right\rangle\left|\phi_{\pm}\left(\theta_1\right)\right\rangle\left|G_0^A\right\rangle\left|G_0^C\right\rangle = \left|\alpha_{\pm}\right\rangle\sum_{i=1}^{h}\sum_{k=1}^{l}\sqrt{\eta_{ik}}\left|G_i^A\right\rangle\left|G_k^C\right\rangle. \tag{3.9}$$

Operation H use only local operations and classical communications which cannot enhance the entanglement. Obviously no entanglement exists in the left side of Eq. (3.9), but the right side is an entangled state between particle A, C_j. Thus such process H does not exist.

However, such process always can be implemented with no-zero probability. In fact, the marginal density operator of the two states $|\alpha_{\pm}\rangle$ are different, so Alice and Clair can discriminate the two states in their system using generalized measurement and then construct states $\left|\phi_{+}\left(\theta_{N-j}\right)\right\rangle\left|\phi_{+}\left(\theta_1\right)\right\rangle$ or $\left|\phi_{-}\left(\theta_{N-j}\right)\right\rangle\left|\phi_{-}\left(\theta_1\right)\right\rangle$ according to measurement results. Denote the optimal probability as μ_j, overall probability to implement $D_j\left(\theta_{N-j},\theta_1\right)$ can be represented as $\frac{1+\mu_j}{2}$. We may also change the form of the three-particle entangled to make the clone optimal. But similar proof as above shows that the probability is always less than 1. However, consider current experiment technology, only two Bell basis $|\Psi^{\pm}\rangle$ of the four can be identified by interferometric schemes, with the others $|\Phi^{\pm}\rangle$ giving the same detection signal [10, 12], so we only consider $|\Psi^{\pm}\rangle$ in our protocol.

After Alice obtain the state $\left|\phi_{\pm}\left(\theta_{N-j}\right)\right\rangle_A$, she take it as the input states $\left|\phi_{\pm}\left(\theta_{N-j}\right)\right\rangle_X$ and use another three-particle entangled state $\left|\psi^{j+1}\right\rangle$ to obtain the states $\left|\phi_{\pm}\left(\theta_{N-(j+1)}\right)\right\rangle_A\left|\phi_{\pm}\left(\theta_1\right)\right\rangle_{C_{j+1}}$ between Alice and C_{j+1}, etc,. In the last process, if Alice wishes to transmit the copies to the associates C_{N-1} and C_N, the system A should be on the side C_N. With the series transformations, the associates C_1, C_2,..., C_N obtain the states $\left|\phi_{\pm}\left(\theta_1\right)\right\rangle_{C_j}$ respectively and they finish the telecloning process.

In the following, we show how to prepare the three-particle entangled state $\left|\psi^j\right\rangle$ represented in Eq. (3.5) using GHZ state by local operations and classical communications. Consider Alice and C_j initially shared a GHZ state $|\xi\rangle_{SAC_j} = \frac{1}{\sqrt{2}}\left(|000\rangle+|111\rangle\right)$, to implement the telecloning process, they must transfer it to the suitable state using only local operations and classical communication. First a local unitary operation $R_y^S\left(\pi/2\right)\otimes R_y^A\left(-\pi/2\right)\otimes R_y^{C_j}\left(-\pi/2\right)$ is used to transfer $|\xi\rangle_{SAC_j}$ to $\left|\xi'\right\rangle_{SAC_j} = \frac{1}{4}\left(\left(|0\rangle-|1\rangle\right)_S\left(|1\rangle+|0\rangle\right)_{AC_j}^{\otimes 2}+\left(|0\rangle+|1\rangle\right)_S\left(|1\rangle-|0\rangle\right)_{AC_j}^{\otimes 2}\right)$. To obtain required states, the local generalized measurement (POVM) is needed, which is described by operators M_m on corresponding system, satisfying the completeness relation $\sum_m M_m^{+}M_m = I$, then the results are sent to other system, who performs an operation ε_m, possible non-unitary, on its system, conditional on the result m.

However, it is difficult to perform the operation ε_m according to classical communication in experiment. In the following, we introduce a method to prepare the initial state by systems S, A and C_j performing local operations respectively without communication. In our protocol, there are two possible final states and both of them can be used for teleclone with same Bell states $|\Psi^{\pm}\rangle$ are measured. Define operations M_{jim} $(i=1,2,3, m=0,1)$ on S, A, and C_j system respectively. They have the following matrix representations in the $|0\rangle$, $|1\rangle$ basis $M_{j10} = \begin{pmatrix}\sin\theta_{N-j+1} & 0\\ 0 & \cos\theta_{N-j+1}\end{pmatrix}$, $M_{j11} = \begin{pmatrix}\cos\theta_{N-j+1} & 0\\ 0 & \sin\theta_{N-j+1}\end{pmatrix}$, $M_{j20} = \begin{pmatrix}\sin\theta_{N-j} & 0\\ 0 & \cos\theta_{N-j}\end{pmatrix}$, $M_{j21} = \begin{pmatrix}\cos\theta_{N-j} & 0\\ 0 & \sin\theta_{N-j}\end{pmatrix}$, $M_{j30} = \begin{pmatrix}\sin\theta_1 & 0\\ 0 & \cos\theta_1\end{pmatrix}$, $M_{j31} = \begin{pmatrix}\cos\theta_1 & 0\\ 0 & \sin\theta_1\end{pmatrix}$. Note that $M_{ji0}^{+}M_{ji0}+M_{ji1}^{+}M_{ji1} = I$, so this defines a generalized measurement on each system, which may be implemented using standard techniques involving only projective measurements and unitary transforms [32]. If we consider a probe P to assist the generalized measurement $M_0 = \begin{pmatrix}\sin\theta & 0\\ 0 & \cos\theta\end{pmatrix}$, $M_1 = \begin{pmatrix}\cos\theta & 0\\ 0 & \sin\theta\end{pmatrix}$, the unitary operator acting on the particle and the probe can be represented as $\begin{pmatrix}R_y\left(-\pi+2\theta\right) & 0\\ 0 & R_y\left(-2\theta\right)\end{pmatrix}$ on the basis $\left\{\left|0P_0\right\rangle,\left|0P_1\right\rangle,\left|1P_0\right\rangle,\left|1P_1\right\rangle\right\}$, where $R_y\left(\theta\right) = \begin{pmatrix}\cos\frac{\theta}{2} & \sin\frac{\theta}{2}\\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2}\end{pmatrix}$ is a rotation by θ around $\hat{y}$. If the measurement result gives $m=1$ for a system, then a rotation σ_x is performed on this system. Let $|\xi_{kpt}\rangle_{SAC_j}$ denote the state after the measurement and local σ_x, given that outcome k, p, t occurred for A, C_j, S system respectively. Then

$$|\xi_{kpt}\rangle_{SAC_j}=\begin{cases}\frac{1}{\sqrt{2}}\left(|0\rangle_S\left|\varphi_1^j\right\rangle_{AC_j}-|1\rangle_S\left|\varphi_0^j\right\rangle_{AC_j}\right),\\ \kappa\left(\frac{t_j}{h_j}|0\rangle_S\left|\varphi_0^j\right\rangle_{AC_j}-\frac{h_j}{t_j}|1\rangle_S\left|\varphi_1^j\right\rangle_{AC_j}\right),\end{cases}\tag{3.10}$$

where $\kappa=\sqrt{\frac{1-\cos^2 2\theta_{N-j+1}}{2(1+\cos^2 2\theta_{N-j+1})}}$, the two states are corresponding to $(-1)^{k+p+t}=\pm1$, respectively. The probability to obtain the first state is $p_1=\frac{\sin^2 2\theta_{N-j+1}}{2}$ and the second is $p_2=\frac{1+\cos^2 2\theta_{N-j+1}}{2}$. The first state in Eq. (3.10) is exactly the state in Eq. (3.5) and the second state can also be used for teleclone. In fact, the combined states of systems $XASAC_j$ can be rewritten in a form that singles out the Bell basis of qubit X and S as

$$\begin{aligned}&\left|\psi^{\pm j}\right\rangle'_{XSAC_j} &(3.11)\\ &=\mp\frac{\kappa}{\sqrt{2}}|\Psi^-\rangle_{XS}\left(h_j\left|\varphi_1^j\right\rangle_{AC_j}\pm t_j\left|\varphi_0^j\right\rangle_{AC_j}\right)\\ &\pm\frac{\kappa}{\sqrt{2}}|\Psi^+\rangle_{XS}\left(h_j\left|\varphi_1^j\right\rangle_{AC_j}\mp t_j\left|\varphi_0^j\right\rangle_{AC_j}\right)\\ &+\frac{\eta}{\sqrt{2}}|\Phi^-\rangle_{XS}\left(h_j^3\left|\varphi_1^j\right\rangle_{AC_j}\pm t_j^3\left|\varphi_0^j\right\rangle_{AC_j}\right)\\ &-\frac{\eta}{\sqrt{2}}|\Phi^+\rangle_{XS}\left(h_j^3\left|\varphi_1^j\right\rangle_{AC_j}\mp t_j^3\left|\varphi_0^j\right\rangle_{AC_j}\right)\end{aligned}$$

where $\eta=\frac{2\kappa}{\sin 2\theta_{N-j+1}}$. Obviously the first two terms can be transferred to the target states using same unitary operations as that in Eq. (3.6). $h_j^3\left|\phi_0^j\right\rangle_{AC_j}\pm t_j^3\left|\phi_1^j\right\rangle_{AC_j}=|\phi_\pm(\theta_{N-j})\rangle|\phi_\pm(\theta_1)\rangle+\cos 2\theta_{N-j+1}|\phi_\mp(\theta_{N-j})\rangle|\phi_\mp(\theta_1)\rangle$ and also can be transferred to the disentangled target states with no-zero probability similar as $|\alpha_\pm\rangle$.

The probabilistic quantum clone process via GHZ state is illustrated in Figure 3 for the case $M=1$, $N=2$.

Figure 3

Let us compare the efficiency of above telecloning process and that using Tele-CNOT gates [7]. To complete a Tele-CNOT operation, two GHZ states and three Bell basis measurement are need, which yields 1/8 probability. Performing a $D_j(\theta_{N-j},\theta_1)$ operation needs three CNOT gates [26], that is, Alice only has probability of $\frac{1}{512}$ to succeed. While our protocol use one GHZ states and yields the probability

$$\begin{aligned}p=p_1\times\frac{1}{2}+p_2\times\kappa^2&=\frac{\sin^2 2\theta_{N-j+1}}{2}\\ &=\frac{1-\cos^{2(N-j+1)}2\theta}{2}.\end{aligned}\tag{3.12}$$

When θ is not too small, the success probability is not too low. If we do not consider the preparation of three-particle entanglement states, the efficiency of Tele-$D_j(\theta_{N-j},\theta_1)$ is 50%, which is exactly the efficiency of Bell measurement. If we have enough GHZ states, we can prepare enough required three-particle entangled states. In the initial information compress process, we adopt the $M\times(1\to N)$ cloning strategy. Using this strategy, more than one $|\phi_\pm(\theta_N)\rangle$ can be obtained. So if the Tele-$D_j(\theta_{N-j},\theta_1)$ operation fails to one $|\phi_\pm(\theta_N)\rangle$, we have chance to use another and increase the success probability. The overall cloning probability of our protocol (not include that in states preparation) can be represented as

$$P=\sum_{k=1}^{M}C_M^k\gamma_{1N}^k(1-\gamma_{1N})^{M-k}\left(1-\left(1-\left(\frac{1}{2}\right)^{N-1}\right)^k\right)\tag{3.13}$$

Obviously P decreases with the increase of N. So in practice we often adopt $1\to2$ cloning strategy.

Such telecloning process can also be accomplished using a multiparticle entangled state, just similar as that has been shown in [29, 30]. The quality of our method is that only three-particle entanglement is used. In this scheme, we use local generalized measurements, Bell basis measurement to avoid the interactions between particles, so it may be feasible in current experiment condition.

IV. SUMMARY

In summary, we have presented a probabilistic quantum clone scheme using GHZ states, Bell basis measurement, single-qubit unitary operations and generalized measurement, all of which are within the reach of current technology. We considered different strategies and propose the concept of *Probability Spectrum* to describe them. For two most important, we show that M entries $1\to N$ cloning process give more copies than one $M\to N$ process at the price of higher probability of failure. Compared to another possible scheme via Tele-CNOT [7] gate, our scheme may be feasible in experiment to *clone the states of one particle to those of two different particles* with higher probability and less GHZ resource.

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Figure Captions:

Figure 1: The expected values of copy number for the two different strategies. Here Solid line, Dashed line, Dotted line and Dash-Dotted line denote $10 \times (1 \rightarrow 20)$, $1 \times (10 \rightarrow 20)$, $2 \times (1 \rightarrow 3)$ and $1 \times (2 \rightarrow 3)$ cloning strategies respectively.

Figure 2: The failure probabilities for the two different strategies. The four kinds of lines represent the same strategies as those in Figure 1.

Figure 3: The logic network of $1 \rightarrow 2$ probabilistic clone via GHZ state. (A) Alice and her associate C_1, C_2 initially share a GHZ state consisting of the qubit S (the port), C_1 and C_2 (outputs, or 'copy qubits'). Alice successfully transforms the initial states $\cos\theta\,|0\rangle_X \pm \sin\theta\,|1\rangle_X$ to $\cos\theta_2\,|0\rangle_X \pm \sin\theta_2\,|1\rangle_X$ if the probe (the location qubit of the photon X) results in $|P_0\rangle$. The parameters $\cos 2\theta_2 = \cos 2\theta$, $\omega = \arccos\sqrt{\frac{(1+\cos^2 2\theta)}{(1+\cos 2\theta)^2}}$. Using the unitary rotation $R_y(\xi)$ and generalized measurement $M(\theta)$, Alice and C_1, C_2 transform GHZ state to the required three-particle entangled state in the form Eq. (3.10). Then Alice performs a Bell measurement of the port S along with 'input' qubit X and has 25% probability to obtain $|\Psi^-\rangle$ or $|\Psi^+\rangle$ respectively; subsequently, the receivers C_1 and C_2 do no operation or σ_x rotations on the output qubits, obtaining two perfect quantum clones.

(B) The implementation of generalized measurement $M(\theta)$ in (A). The location qubit of the photon is adopted as the probe P.